ERRATA

Erratum: Multifractal analysis of the atomic spectral line series [Phys. Rev. E 54, 2431 (1996)]

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We have found an error in the code used to calculate the Rényi scaling indices $D(q)$ for the spectral line series in our paper. For the multifractal sets used to test the program and to illustrate the method in Sec. II A, the effect of the error in the scaling indices is completely negligible. For this reason we found a good agreement between the experimental curves $D(q)$ [as well as $f(\alpha)$] and the theoretically expected ones. For the line series of argon and cobalt, the differences between the results presented in the paper and the ones obtained once the problem has been solved are also small, and of the same order of the error bars estimated by slightly varying the range of scales used to fit the power laws.

However, for the line series of the hydrogen atom the error had conspicuous consequences. We have found that the results for $D(q)$ and $f(\alpha)$ are not really meaningful. To clarify the point it is useful to distinguish two cases: $q=0$ and $q\neq 0$. As has been stated in the paper, it is rather straightforward to show that the capacity dimension of the nonfractal set of points in the unit interval $\{1/n\}_{n=1}^{\infty}$ has box-counting dimension $D(0)=1/2$. As Górski [1] has pointed out, the dimension $D(0)$ of the hydrogen line series has to be the same as the dimension of the compact set $\{1/n^2\}_{n=1}^{\infty}$, and in this case the analytical expected value is 1/3. Nevertheless, the numerical estimation of the box-counting dimension of the set $\{1/n^2\}_{n=1}^{1000}$ and the Balmer and Paschen series are not completely equivalent. In the latter case, a scale-dependent behavior of the exponent $D(0)$ is observed as is illustrated in Fig. 1. $D(0)$ takes a value of 0.51 in the range $[2^{-10},2^{-1}]$ (the range used in our paper), and a value of 0.34 within the range $[2^{-20},2^{-10}]$, while for $\{1/n^2\}_{n=1}^{1000}$ a single straight line can be fitted to the whole range $[2^{-25},2^{-1}]$, as pointed out by Górsky. We have also plotted the corresponding curve for the set $\{1/n\}_{n=1}^{1000}$; here again a single straight line with slope $\simeq -0.5$ can be fitted to the data, but within a smaller range of scales: $[2^{-17},2^{-1}]$. For smaller partitions, the number of occupied cells remains constant because the side of the box is smaller than the spacing between the data. We must conclude that the correct value of $D(0)$ for the line series of the hydrogen atom is 1/3. The choice of the scaling range in our paper was good for all the analyzed cases except for this one. As we have shown in the plot, the slope corresponding to the smaller scales where a log-log least-squares fit of $N(r)$ vs r can be performed provides a value of $D(0)\simeq 1/3$.

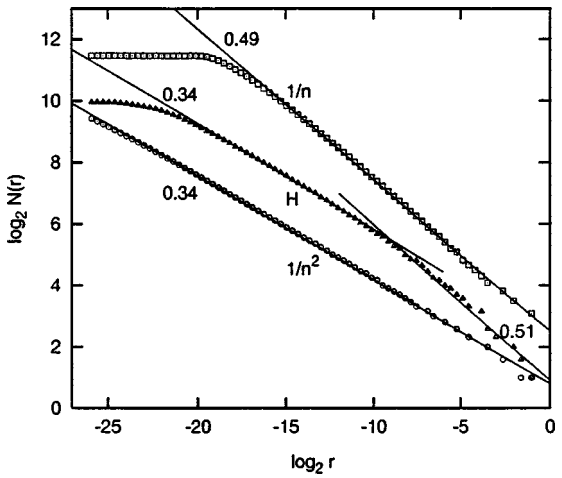

FIG. 1. Log-log plot of the number of occupied cells $N(r)$ as a function of the scale r for the Balmer and Paschen series (H) (996 lines) and the sets $\{1/n^2\}_{n=1}^{1000}$ and $\{1/n\}_{n=1}^{1000}$. For the latter case, the plot has been shifted vertically to avoid confusion. The values of $D(0)$ obtained from the slopes of the least-squares fits are displayed.

For values of $q\neq 0$, and the number of points $N\sim 1000$, we do not obtain meaningful values of $D(q)$ by means of the standard box-counting algorithm. The regression coefficients are unacceptable in any significant range of scales, therefore Figs. 2(c) and 2(d) and the numbers extracted from them are not valid.

The multiplicative model was an example of an inhomogeneous distribution on a nonfractal support being a multifractal. The hydrogen spectral line series, and in general the inverse power series, are clear examples of nonfractal sets having a noninteger box-counting dimension.

We want to thank A. Z. Górski for having pointed out the problem in our paper regarding the value of $D(0)$ for the Balmer series.

[1] A. Z. Górski, e-print chao-dyn/9804034.

